Lara, Danielle; **Marchesi, Simone**; **Martins, Renato Vidal**
Curves with canonical models on scrolls. (English) Zbl 1357.14040
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Throughout, let C be a curve (i.e., an integral, complete, one-dimensional scheme) over an algebraically closed field of arithmetic genus g. Let $C' \subseteq \mathbb{P}^{g-1}$ be its canonical model which is defined by the global sections of the dualizing sheaf of C. It is well-known so far that properties on trigonal Gorenstein curves can be deduced whenever its canonical model is contained in a surface scroll; e.g. [*K.-O. Stöhr*, J. Pure Appl. Algebra 135, No. 1, 93–105 (1999; Zbl 0940.14018)], [*R. Rosa* and *K.-O. Stöhr*, J. Pure Appl. Algebra 174, No. 2, 187–205 (2002; Zbl 1059.14038)].

In this paper the authors study the case where C is non-Gorenstein and C' is contained in a scroll surface. Here the concepts "nearly Gorenstein" and "arithmetically normal" become relevant according respectively to Theorems 5.10 and 4 in [*S. L. Kleiman* and *R. V. Martins*, Geom. Dedicata 139, 139–166 (2009; Zbl 1172.14019)]. Moreover, as looking at for examples, they consider rational monomial curves and show that for such a curve its canonical model is contained in a scroll surface if and only if the curve is trigonal. This leads to the question when a nonhyperelliptic curve can be characterized by its canonical model; in fact, this is worked out for the case of a nonhyperelliptic curve with at most one unibranched singular point. Finally they generalize some results in [*F.-O. Schreyer*, Math. Ann. 275, 105–137 (1986; Zbl 0578.14002)].

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MSC:

14H20	Singularities of curves, local rings
14H45	Special algebraic curves and curves of low genus
14H51	Special divisors on curves (gonality, Brill-Noether theory)

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non-Gorenstein curves; canonical model; trigonal non-Gorenstein curve; scrolls

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References:

[1] 1. E. Arbarello, M. Cornalba, P. A. Griffiths and J. Harris, Geometry of Algebraic Curves (Springer-Verlag, 1985). genRefLink(16, 'S0129167X16500452BIB001', '10.1007%252F978-1-4757-5323-3'); · Zbl 0559.14017

[2] 2. E. Arrondo, A home-made Hartshorne-Serre correspondence, preprint (2006), arXiv:math/0610015v1 [math.AG]. · Zbl 1133.14046

[3] 3. D. W. Babbage, A note on the quadrics through a canonical curve, J. London Math. Soc.14 (1939) 310-315. genRefLink(16, 'S0129167X16500452BIB003', '10.1112%252Fjlms%252Fs1-14.4.310'); · Zbl 0026.34701

[4] 4. V. Barucci and R. Fröberg, One-dimensional almost gorenstein rings, J. Algebra188 (1997) 418-442. genRefLink(16, 'S0129167X16500452BIB004', '10.1006%252Fjabr.1996.6837'); genRefLink(128, 'S0129167X16500452BIB004', 'A1997WN01200003');

[5] 5. M. Brundu and G. Sacchiero, Stratification of the moduli space of fourgonal curves, Proc. Edinb. Math. Soc.57(3) (2014) 631-686. genRefLink(16, 'S0129167X16500452BIB005', '10.1017%252FS001309151300062X'); genRefLink(128, 'S0129167X16500452BIB005', '000341567000004');

[6] 6. G. Casnati and T. Ekedahl, Covers of algebraic varieties. I. A general structure theorem, covers of degree 3,4 and Enriques surfaces, J. Algebraic Geom.5(3) (1996) 439-460. · Zbl 0866.14009

[7] 7. M. Coppens, Free linear systems on integral Gorenstein curves, J. Algebra145 (1992) 209-218. genRefLink(16, 'S0129167X16500452BIB007', '10.1016%252F0021-8693%252892%252990186-P'); genRefLink(128, 'S0129167X16500452BIB007', 'A1992HC18500014');

[8] 8. D. Eisenbud and J. Harris, On varieties of minimal degree, Proc. Symp. Pure Math.46 (1987) 3-13. genRefLink(16, 'S0129167X16500452BIB008', '10.1090%252Fpspum%252F046.1%252F927946'); · Zbl 0646.14036

[9] 9. F. Enriques, Sulle curve canoniche di genera p cello spazio a p dimensioni, Rend. Accad. Sci. Ist. Bologna23 (1919) 80-82.

[10] 10. L. Feital and R. V. Martins, Gonality of non-Gorenstein curves of genus five, Bull. Braz. Math. Soc.45(4) (2014) 1-22. genRefLink(16, 'S0129167X16500452BIB010', '10.1007%252Fs00574-014-0067-5'); · Zbl 1308.14029

[11] 11. R. Hartshorne, Algebraic Geometry (Springer-Verlag, 1977). genRefLink(16, 'S0129167X16500452BIB011', '10.1007%252F978-

1-4757-3849-0');

[12] 12. S. L. Kleiman and R. V. Martins, The canonical model of a singular curvee, Geom. Dedicata139 (2009) 139-166. genRefLink(16, 'S0129167X16500452BIB012', '10.1007%252Fs10711-008-9331-4'); genRefLink(128, 'S0129167X16500452BIB012', '000263790400011');

[13] 13. D. Lara, Curvas com modelos canônicos em scrolls, Ph.D. thesis, UFMG (2014).

[14] 14. R. V. Martins, On trigonal non-Gorenstein curves with zero Maroni invariant, J. Algebra275 (2004) 453-470. genRefLink(16, 'S0129167X16500452BIB014', '10.1016%252Fj.jalgebra.2003.10.033'); genRefLink(128, 'S0129167X16500452BIB014', '000221074300001');

[15] 15. R. V. Martins, Trigonal non-Gorenstein curves, J. Pure Appl. Algebra209 (2007) 873-882. genRefLink(16, 'S0129167X16500452BIB015', '10.1016%252Fj.jpaa.2006.08.010'); genRefLink(128, 'S0129167X16500452BIB015', '000244819700021');

[16] 16. R. M. Miró-Roig, The representation type of rational normal scrolls, Rend. Circ. Mat. Palermo62 (2012) 153-164. genRefLink(16, 'S0129167X16500452BIB016', '10.1007%252Fs12215-013-0113-y'); · Zbl 1268.14014

[17] 17. M. Noether, Über die invariante darstellung algebraicher funktionen, Math. Ann.17 (1880) 263-284. genRefLink(16, 'S0129167X16500452BIB017', '10.1007%252FBF01443474');

[18] 18. K. Petri, Über die invariante darstellung algebraischer funktionen eiener veränderlichen, Math. Ann.88 (1922) 242-289. genRefLink(16, 'S0129167X16500452BIB018', '10.1007%252FBF01579181'); · Zbl 49.0264.02

[19] 19. M. Reid, Chapters on algebraic surfaces, preprint (1996), arXiv:alg-geom/9602006v1.

[20] 20. R. Rosa and K.-O. Stöhr, Trigonal Gorenstein curves, J. Pure Appl. Algebra174 (2002) 187-205. genRefLink(16, 'S0129167X16500452BIB020', '10.1016%252FS0022-4049%252802%252900122-6'); genRefLink(128, 'S0129167X16500452BIB020', '000178014700006');

[21] 21. M. Rosenlicht, Equivalence relations on algebraic curves, Ann. Math.56 (1952) 169-191. genRefLink(16, 'S0129167X16500452BIB021', '10.2307%252F1969773'); genRefLink(128, 'S0129167X16500452BIB021', 'A1952UL93200011');

[22] 22. F. Sakai, Weil divisors on normal surfaces, Duke Math. J.51 (1984) 877-887. genRefLink(16, 'S0129167X16500452BIB022', '10.1215%252FS0012-7094-84-05138-X'); genRefLink(128, 'S0129167X16500452BIB022', 'A1984AAC1400004');

[23] 23. F.-O. Schreyer, Syzygies of canonical curves and special linear series, Math. Ann.275 (1986) 105-137. genRefLink(16, 'S0129167X16500452BIB023', '10.1007%252FBF01458587'); genRefLink(128, 'S0129167X16500452BIB023', 'A1986D357400009');

[24] 24. K.-O. Stöhr, On the poles of regular differentials of singular curves, Bol. Soc. Brasil. Mat.24 (1993) 105-135. genRefLink(16, 'S0129167X16500452BIB024', '10.1007%252FBF01231698'); · Zbl 0788.14020

[25] 25. K.-O. Stöhr, Hyperelliptic Gorenstein curves, J. Pure Appl. Algebra135 (1999) 93-105. genRefLink(16, 'S0129167X16500452BIB025', '10.1016%252FS0022-4049%252897%252900124-2'); genRefLink(128, 'S0129167X16500452BIB025', '000078247600007');

[26] 26. K.-O. Stöhr and P. Viana, Weierstrass gap sequences and moduli varieties of trigonal curves, J. Pure Appl. Algebra81 (1992) 63-82. genRefLink(16, 'S0129167X16500452BIB026', '10.1016%252F0022-4049%252892%252990135-3'); genRefLink(128, 'S0129167X16500452BIB026', 'A1992JG90300006');