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A matricial view of the Karpelevič theorem. (English) Zbl 1359.15029

Summary: The question of the exact region in the complex plane of the possible single eigenvalues of all $n$-by-$n$ stochastic matrices was raised by Kolmogorov in 1937 and settled by Karpelevič in 1951 after a partial result by Dmitriev and Dynkin in 1946. The Karpelevič result is unwieldy, but a simplification was given by D. Ž. Đoković in [Linear Algebra Appl. 142, 173–193 (1990; Zbl 0726.15016)] and H. Ito in [Linear Algebra Appl. 267, 241–246 (1997; Zbl 0888.15006)]. The Karpelevič region is determined by a set of boundary arcs each connecting consecutive roots of unity of order less than $n$. It is shown here that each of these arcs is realized by a single, somewhat simple, parameterized stochastic matrix. Other observations are made about the nature of the arcs and several further questions are raised. The doubly stochastic analog of the Karpelevič region remains open, but a conjecture about it is amplified.

MSC:
15B51 Stochastic matrices
15A18 Eigenvalues, singular values, and eigenvectors
15A29 Inverse problems in linear algebra

Keywords: stochastic matrix; doubly stochastic matrix; Karpelevič arc; Karpelevič region; Ito polynomial; realizing matrix

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References:
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