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Geometry of infinite planar maps with high degrees. (English) [Zbl 1360.05151]

Summary: We study the geometry of infinite random Boltzmann planar maps with vertices of high degree. These correspond to the duals of the Boltzmann maps associated to a critical weight sequence \((q_k)_{k \geq 0}\) for the faces with polynomial decay \(k^{-a}\) with \(a \in (\frac{3}{2}, \frac{5}{2})\) which have been studied by J.-F. Le Gall and G. Miermont [Ann. Probab. 39, No. 1, 1–69 (2011; Zbl 1204.05088)] as well as by G. Borot et al. [J. Phys. A, Math. Theor. 45, No. 4, Article ID 045002, 38 p. (2012; Zbl 1235.82026)]. We show the existence of a phase transition for the geometry of these maps at \(a = 2\). In the dilute phase corresponding to \(a \in (2, \frac{5}{2})\) we prove that the volume of the ball of radius \(r\) (for the graph distance) is of order \(r^d\) with \(d = (a - \frac{1}{2})/(a - 2)\), and we provide distributional scaling limits for the volume and perimeter process. In the dense phase corresponding to \(a \in (\frac{3}{2}, 2)\) the volume of the ball of radius \(r\) is exponential in \(r\). We also study the first-passage percolation (fpp) distance with exponential edge weights and show in particular that in the dense phase the fpp distance between the origin and \(\infty\) is finite. The latter implies in addition that the random lattices in the dense phase are transient. The proofs rely on the recent peeling process introduced by T. Budd [Electron. J. Comb. 23, No. 1, Research Paper P1.28, 37 p. (2016; Zbl 1331.05192)] and use ideas of N. Curien and J.-F. Le Gall [Ann. Inst. Henri Poincaré, Probab. Stat. 53, No. 1, 322–357 (2017; Zbl 1358.05255)] in the dilute phase.

MSC:

05C80 Random graphs (graph-theoretic aspects)
05C12 Distance in graphs
60G52 Stable stochastic processes
05C81 Random walks on graphs

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random planar map; scaling limit; peeling process; graph distance; stable processes

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