The classification of edges and the change in multiplicity of an eigenvalue of a real symmetric matrix resulting from the change in an edge value. (English) Zbl 1360.15014 Spec. Matrices 5, 51-60 (2017).

Summary: We take as given a real symmetric matrix \( A \), whose graph is a tree \( T \), and the eigenvalues of \( A \), with their multiplicities. Each edge of \( T \) may then be classified in one of four categories, based upon the change in multiplicity of a particular eigenvalue, when the edge is removed (i.e. the corresponding entry of \( A \) is replaced by 0). We show a necessary and sufficient condition for each possible classification of an edge. A special relationship is observed among 2-Parter edges, Parter edges and singly Parter vertices. Then, we investigate the change in multiplicity of an eigenvalue based upon a change in an edge value. We show how the multiplicity of the eigenvalue changes depending upon the status of the edge and the edge value. This work explains why, in some cases, edge values have no effect on multiplicities. We also characterize, more precisely, how multiplicity changes with the removal of two adjacent vertices.

MSC:

15A18 Eigenvalues, singular values, and eigenvectors
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
05C05 Trees
15B57 Hermitian, skew-Hermitian, and related matrices

Keywords:
edges; eigenvalues; graph; matrix entries; multiplicity; real symmetric matrix; tree

Full Text: DOI

References:
3 C.R. Johnson, A.Leal Duarte, The maximum multiplicity of an eigenvalue in a matrix whose graph is a tree, Linear Multilinear Algebra 46 (1999), 139-144. - Zbl 0929.15005
6 C.R. Johnson, A. Leal-Duarte and C.M. Saiago, The change in eigenvalue multiplicity associated with perturbation of a diagonal entry, Linear Multilinear Algebra 60 (2012), no.5 525-532. - Zbl 1247.15007
8 G.Wiener, Spectral multiplicity and splitting results for a class of qualitative matrices, Linear Algebra Appl. 61 (1984), 15-29. - Zbl 0549.15004

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