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Categorically proper homomorphisms of topological groups. (English) Zbl 1360.22003

D. N. Dikranjan and V. V. Uspenskij [J. Pure Appl. Algebra 126, No. 1–3, 149–168 (1998; Zbl 0887.22001)] call a (Hausdorff) topological group $G$ c-compact if the projection $G \times K \to K$ maps closed subgroups onto closed subgroups for every Hausdorff topological group $K$. Surely compact groups are c-compact. Dikranjan and Uspenskij [loc. cit.] show that for soluble groups the converse is true. In general the converse is not true. This was proved by A. A. Klyachko et al. [Topology Appl. 160, No. 16, 2104–2120 (2013; Zbl 1285.22003)].

In this paper, the authors study the related notion of c-properness in the category of (non-necessarily Hausdorff) topological groups. In Section 2 categorically closed, proper and perfect maps are considered. In particular it is shown:

For a topological group $G$ the following assertions are equivalent: (i) $G$ is c-compact; (ii) every $f : G \to H$ with $H$ Hausdorff is c-proper; (iii) there is a c-proper $f : G \to H$ with $H$ c-compact.

In Section 3 categorically and homomorphically complete objects and morphisms are studied. In particular it is shown:

For a topological group $G$ the following assertions are equivalent: (i) $G$ is c-complete; (ii) every $f : G \to H$ with $H$ Hausdorff is c-complete; (iii) there is a c-complete $f : G \to H$ with $H$ c-compact. These equivalences remain valid if "c-complete" is traded for "$h$-complete" everywhere.

In Section 4 c-proper maps resp. $h$-complete maps are characterized by special filters. By using results from this section the following theorem is proved in Section 5:

Let $(f_i : G_i \to H_i)_{i \in I}$ be a family of c-proper (c-perfect; $h$-complete) maps with every $H_i$ Hausdorff. Then their product $\prod_{i \in I} f_i : \prod G_i \to \prod H_i$ is also c-proper (c-perfect; $h$-complete).

This theorem implies the following known result: The direct product of any family of c-compact groups is c-compact. Likewise, $h$-completeness is preserved under taking direct products of topological groups.

Reviewer: Dieter Remus (Hagen)

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54H11 Topological groups (topological aspects)
18B99 Special categories

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