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On the Narasimhan-Seshadri correspondence for real and quaternionic vector bundles. (English) Zbl 1360.30037

J. Differ. Geom. 105, No. 1, 119-162 (2017).

Summary: Let (M,σ) be a compact Klein surface of genus $g \geq 2$ and let E be a smooth Hermitian vector bundle on M. Let τ be a Real or Quaternionic structure on E and denote respectively by $\mathcal{G}_{\mathbb{C}}^{\tau}$ and $\mathcal{G}_{E}^{\tau}$ the groups of complex linear and unitary automorphisms of E that commute to τ. In this paper, we study the action of $\mathcal{G}_{\mathbb{C}}^{\tau}$ on the space $\mathcal{A}_{E}^{\tau}$ of τ-compatible unitary connections on E and show that the closure of a semi-stable $\mathcal{G}_{\mathbb{C}}^{\tau}$-orbit contains a unique $\mathcal{G}_{E}^{\tau}$-orbit of projectively flat connections. We then use this invariant-theoretic perspective to prove a version of the Narasimhan-Seshadri correspondence in this context: S-equivalence classes of semi-stable Real and Quaternionic vector bundes are in bijective correspondence with equivalence classes of certain appropriate representations of orbifold fundamental groups of Real Seifert manifolds over the Klein surface (M,σ).

MSC:

30F50 Klein surfaces
32L05 Holomorphic bundles and generalizations

Keywords:

Klein surfaces; Hermitian vector bundles

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