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Summary: We study the problem of maximizing a monotone non-decreasing function $f$ subject to a matroid constraint. Fisher, Nemhauser and Wolsey have shown that, if $f$ is submodular, the greedy algorithm will find a solution with value at least $\alpha$ of the optimal value under a uniform matroid constraint. If $f$ is not submodular, the greedy algorithm can achieve a $(1-1/e)$-approximation under a uniform matroid constraint. In this paper, we show that the greedy algorithm can find a solution with value at least $\alpha$ of the optimum value for a general monotone non-decreasing function with a general matroid constraint, where $\mu = \alpha$, if $0 \leq \alpha \leq 1$; $\mu = \frac{\alpha^k(1-\alpha^k)}{K(1-\alpha)}$ if $\alpha > 1$; here $\alpha$ is a constant representing the "elemental curvature" of $f$, and $K$ is the cardinality of the largest maximal independent sets. We also show that the greedy algorithm can achieve a $1 - (\frac{\alpha^k(1-\alpha^k)}{1+\alpha^k+\cdots+\alpha^{k-1}})^k$ approximation under a uniform matroid constraint. Under this unified $\alpha$-classification, submodular functions arise as the special case $0 \leq \alpha \leq 1$.

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monotone submodular set function; matroid; approximation algorithm

References:


