Crasmareanu, Mircea; Ida, Cristian; Popescu, Paul
An 1-differentiable cohomology induced by a vector field. (English) Zbl 1361.58001
J. Lie Theory 26, No. 4, 911-926 (2016).

Let $M$ be a smooth manifold, let $\Omega^\bullet(M)$ denote differential forms and let $X$ be a vector field on $M$. The Lie derivative along $X$ is a cochain map $L_X : (\Omega^\bullet(M), d) \to (\Omega^\bullet(M), d)$. Hence the mapping cone construction (from Homological Algebra) applies, and the authors can define a new cochain complex $(\tilde{\Omega}^\bullet(M), \tilde{d}_X)$, where $\tilde{\Omega}^\bullet(M) = \Omega^\bullet(M) \oplus \Omega^{\bullet-1}(M)$, and $\tilde{d}_X(\phi, \psi) = (d\phi, L_X\phi - \psi)$, for all $(\phi, \psi) \in \tilde{\Omega}^\bullet(M)$. The paper discusses some elementary properties of $(\tilde{\Omega}^\bullet(M), \tilde{d}_X)$, also in relation with symplectic, Riemannian and complex geometry. Unfortunately, cohomology of $(\tilde{\Omega}^\bullet(M), \tilde{d}_X)$ is (canonically) isomorphic to $H^\bullet_{dR}(M) \oplus H^{\bullet-1}_{dR}(M)$. In particular, it is not a new invariant of vector field $X$, as it does not contain any information on it.

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MSC:
- 58A10 Differential forms in global analysis
- 14F40 de Rham cohomology and algebraic geometry
- 53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)
- 58A12 de Rham theory in global analysis

Keywords:
de Rham cohomology; 1-differentiable form; Lie derivative; vector field; harmonic form

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