For a given topological toric manifold $M$, someone is able to obtains its characteristic map, enabling the notation $M = M(K, \lambda)$. A characteristic map of dimension $n$ is defined as the pair $(K, \lambda)$ of a face complex $K$ of dimension $\leq n - 1$ and a map $\lambda : V(K) \rightarrow \mathbb{Z}^n$ so that $\{\lambda(i) \mid i \in \sigma\}$ is a linearly independent set over $\mathbb{R}$ for any face $\sigma$ of $K$, where $V(K)$ is the vertex set of $K$. There is a classical operation of simplicial complexes which is called the simplicial wedge operation. In this operation from a simplicial complex $K$ with $m$ vertices and for a fixed vertex $v$, is defined a simplicial complex of $m + 1$ vertices, which is called the wedge of $K$ at $v$ and it is denoted by $\text{wedge}_v(K)$.

In their main result the authors prove the following: Let $K$ be a fan-like simplicial sphere and $v$ a given vertex of $K$. Let $(\text{wedge}_v(K), \lambda)$ be a characteristic map and let $v_1$ and $v_2$ be the two new vertices of $\text{wedge}_v(K)$ created from the wedging, where $\{\lambda(v_1), \lambda(v_2)\}$ is a unimodular set. Then $\lambda$ is uniquely determined by the projections $\text{Proj}_{v_1}\lambda$ and $\text{Proj}_{v_2}\lambda$. In other words, they prove that in order to find all toric objects $(\text{wedge}_v(K), \mu)$ it suffices to determine all toric objects $(K, \lambda)$.

In the second main part of the article, the authors complete the classification of toric manifolds and topological toric manifolds with Picard number at most three. We remind that if $m$ is the number of rays of a complete non-singular fan of dimension $n$ and $K$ is the corresponding simplicial complex of dimension $n - 1$, then the Picard number of $K$ is defined as $m - n$. They study several other applications which are occuring from the above results and from the techniques that they develop.

Reviewer: Christos Tatakis (Mitilini)

MSC:

| 14M25 | Toric varieties, Newton polyhedra, Okounkov bodies |
| 52B20 | Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry) |
| 52B35 | Gale and other diagrams |

Keywords:

toric variety; projective toric variety; Gale diagram; simplicial wedge; topological toric manifold; real topological toric manifold; quasitoric manifold; small cover; real toric variety

Software:

Convex

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References:
