Liu, Gongxiang; Van Oystaeyen, Fred; Zhang, Yinhuo
Representations of the small quasi-quantum group $\text{Qu}_q(\mathfrak{sl}_2)$. (English) Zbl 1364.17020

The small quantum group $(u_q)(\mathfrak{sl}_2)$ was introduced by G. Lusztig [J. Am. Math. Soc. 3, No. 1, 257–296 (1990; Zbl 0695.16006)]. The first author of the paper under review introduced a quasi-Hopf version denoted $(\text{Qu}_q)(\mathfrak{sl}_2)$ [Math. Res. Lett. 21, No. 3, 585–603 (2014; Zbl 1315.16029)], where q is a primitive $(n^2)$-th root of unity, and showed that for odd $n$, it is twisted equivalent to the Hopf algebra $(u_q)(\mathfrak{sl}_2)$, but for even $n$ it is not. The paper under review studies the representations of $(\text{Qu}_q)(\mathfrak{sl}_2)$ for even $n$. They restrict to $n$ divisible by 4, but remark on the general case. The authors give a complete list of non-isomorphic indecomposable modules. There are no Steinberg modules (simple projective modules) and the dimensions of all simple modules are odd. All blocks have the same dimension and they are Morita equivalent to one another. The basic algebra of $(\text{Qu}_q)(\mathfrak{sl}_2)$ can be equipped with a Hopf algebra structure, but this does not happen for the basic algebra of $(u_q)(\mathfrak{sl}_2)$. The authors give the decomposition rules of tensor products using a version of the Clebsch-Gordan formula, and determine the Grothendieck ring $K_0$ of $(\text{Qu}_q)(\mathfrak{sl}_2)$.

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