MacLean, Mark S.; **Miklavič, Štefko**

On bipartite distance-regular graphs with exactly one non-thin T-module with endpoint two. (English) Zbl 1365.05078

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Summary: Let Γ denote a bipartite distance-regular graph with diameter $D \geq 4$ and valency $k \geq 3$. Let X denote the vertex set of Γ, and let A denote the adjacency matrix of Γ. For $x \in X$ and for $0 \leq i \leq D$, let $\Gamma_i(x)$ denote the set of vertices in X that are distance i from vertex x. Define a parameter Δ_2 in terms of the intersection numbers by $\Delta_2 = (k-2)(c_3-1)-(c_2-1)p^2_{22}$.

For $x \in X$ let $T = T(x)$ denote the subalgebra of $\mathrm{Mat}_X(\mathbb{C})$ generated by A, E^*_0, E^*_1, ..., E^*_D, where for $0 \leq i \leq D$, E^*_i represents the projection onto the ith subconstituent of Γ with respect to x. We refer to T as the Terwilliger algebra of Γ with respect to x. An irreducible T-module W is said to be thin whenever $\dim(E^*_iW) \leq 1$ for $0 \leq i \leq D$. By the endpoint of an irreducible T-module W we mean $\min\{i \mid E^*_iW \neq 0\}$.

Fix $x \in X$ and assume that Γ has, up to isomorphism, exactly one irreducible T-module W with endpoint 2, and that W is non-thin with $\dim(E^*_2W) = 1$, $\dim(E^*_{D-1}W) \leq 1$ and $\dim(E^*_iW) \leq 2$ for $3 \leq i \leq D$. We prove that for $2 \leq i \leq D$, there exist complex scalars α_i, β_i such that $|\Gamma_{i-1}(x) \cap \Gamma_{i-1}(y) \cap \Gamma_1(z)| = \alpha_i + \beta_i|\Gamma_1(x) \cap \Gamma_1(y) \cap \Gamma_{i-1}(z)|$ for all $y \in \Gamma_2(x)$ and $z \in \Gamma_i(x) \cap \Gamma_i(y)$. Furthermore, we prove $\Delta_2 = 0$ and either $D = 5$ or $c_2 \in \{1, 2\}$. We show there exist integers $3 \leq f \leq \ell \leq D-2$ such that $\dim(E^*_iW) = 2$ if and only if $f \leq i \leq \ell$.

MSC:

05C12 Distance in graphs

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