Exact sequences $A \to B \to C$ of tensor categories are introduced, relative to some indecomposable exact $A$-module category $\mathcal{M}$. (Here a tensor category means an autonomous monoidal category enriched in the category $\text{Vec}$ of vector spaces over a given field, subject to a few further – mainly finiteness – conditions.) In the case when $\mathcal{M}$ is equal to $\text{Vec}$, the definition in [A. Bruguières and S. Natale, Int. Math. Res. Not. 2011, No. 24, 5644–5705 (2011; Zbl 1250.18005)] is re-obtained; in fact an $\mathcal{M}$-relative exact sequence is an exact sequence $A \to B \to C \boxtimes \text{End}(\mathcal{M})$ in the sense of Bruguières and Natale, where $C \boxtimes \text{End}(\mathcal{M})$ is the Deligne tensor product with the category of right exact endofunctors of $\mathcal{M}$. Allowing for these more general module categories $\mathcal{M}$ instead of $\text{Vec}$, the existence of a fiber tensor functor $A \to \text{Vec}$ no longer follows. That is, $A$ no longer needs to be equivalent to the category of comodules over some Hopf algebra.

Several results due to Bruguières and Natale are extended to this more general setting. For example, relative exact sequences are characterized by the multiplicativity of the Frobenius-Perron dimension. Semisimplicity of the middle term is proven in those relative exact sequences in which all other tensor categories are semisimple.

Even more importantly, the more general setting in the paper under review allows to prove some results which were not available in the situation discussed by Bruguières and Natale. Namely, Deligne tensor products of tensor categories are shown to induce exact sequences in this more general sense. Moreover, the class of such relative exact sequences is proven to be closed under a suitable duality.

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MSC:

18E10 Abelian categories, Grothendieck categories
18D10 Monoidal, symmetric monoidal and braided categories (MSC2010)
18D15 Closed categories (closed monoidal and Cartesian closed categories, etc.)
18D20 Enriched categories (over closed or monoidal categories)
16T05 Hopf algebras and their applications

Keywords:
tensor category; module category; exact sequence

Full Text: DOI arXiv

References:

[2] Bruguières, A.; Natale, S., Central exact sequences of tensor categories, equivariantization and applications - Zbl 1311.18009

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