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Cone avoiding closed sets. (English) Zbl 1369.03101

Summary: We prove that for an arbitrary subtree $T$ of $2^{<\omega}$ with each element extendable to a path, a given countable class $\mathcal{M}$ closed under disjoint union, and any set $A$, if none of the members of $\mathcal{M}$ strongly $k$-enumerate $T$ for any $k$, then there exists an infinite set contained in either $A$ or $\bar{A}$ such that for every $C \in \mathcal{M}$, $C \oplus G$ also does not strongly $k$-enumerate $T$. We give applications of this result, which include:

1. $\mathsf{RT}_2^2$ doesn’t imply $\mathsf{WWKL}_0$;
2. [K. Ambos-Spies et al., J. Symb. Log. 69, No. 4, 1089–1104 (2004; Zbl 1076.03039)] DNR is strictly weaker than $\mathsf{WWKL}_0$;
3. [B. Kjos-Hanssen, Math. Res. Lett. 16, No. 1, 103–110 (2009; Zbl 1179.03061)] for any Martin-Löf random set $A$, either $A$ or $\bar{A}$ contains an infinite subset that does not compute any Martin-Löf random set; etc. We also discuss further generalizations of this result.

MSC:

03B30 Foundations of classical theories (including reverse mathematics)
03F35 Second- and higher-order arithmetic and fragments
03C62 Models of arithmetic and set theory
03D32 Algorithmic randomness and dimension
03D80 Applications of computability and recursion theory
68Q30 Algorithmic information theory (Kolmogorov complexity, etc.)

Keywords:

Ramsey’s theorem; weak weak König’s lemma; Martin-Löf randomness; randomness extraction

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References:
