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Convergence and error propagation results on a linear iterative unfolding method. (English)

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Summary: Unfolding problems often arise in the context of statistical data analysis. Such problematics occur when the probability distribution of a physical quantity is to be measured, but it is randomized (smeared) by some well-understood process, such as a nonideal detector response or a well-described physical phenomenon. In such case it is said that the original probability distribution of interest is folded by a known response function. The reconstruction of the original probability distribution from the measured one is called unfolding. That technically involves evaluation of the nonbounded inverse of an integral operator over the space of L^1 functions, which is known to be an ill-posed problem. For the pertinent regularized operator inversion, we propose a linear iterative formula and provide proof of convergence in a probability theory context. Furthermore, we provide formulae for error estimates at finite iteration stopping order which are of utmost importance in practical applications: the approximation error, the propagated statistical error, and the propagated systematic error can be quantified. The arguments are based on the Riesz-Thorin theorem mapping the original L^1 problem to L^2 space, and subsequent application of ordinary L^2 spectral theory of operators. A library implementation in C of the algorithm along with corresponding error propagation is also provided. A numerical example also illustrates the method in operation.

MSC:

47A52	Linear operators and ill-posed problems, regularization
47N30	Applications of operator theory in probability theory and statistics
65J10	Numerical solutions to equations with linear operators
62-07	Data analysis (statistics) (MSC2010)
65C60	Computational problems in statistics (MSC2010)

Keywords:

unfolding; convergence; error propagation; probability theory; statistics; functional analysis; Riesz-Thorin theorem

Software:

RooUnfold; Libunfold; GSL

Full Text: DOI arXiv

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