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Tractability, hardness, and kernelization lower bound for and/or graph solution. (English)
Zbl 1372.05224

Summary: And/or graphs are well-known data structures with several applications in many fields of computer science, such as Artificial Intelligence, Distributed Systems, Software Engineering, and Operations Research. An and/or graph is an acyclic digraph $G$ containing a single source vertex $s$, where every vertex $v \in V(G)$ has a label $f(v) \in \{\text{and}, \text{or}\}$. In an and/or graph, (weighted) edges represent dependency relations between vertices: a vertex labeled and depends on all of its out-neighbors, while a vertex labeled or depends on only one of its out-neighbors. A solution subgraph $H$ of an and/or graph $G$ is a subdigraph of $G$ containing its source vertex and such that if an and-vertex (resp. or-vertex) is included in $H$ then all (resp. one) of its out-edges must also be included in $H$. In general, solution subgraphs represent feasible solutions of problems modeled by and/or graphs. The optimization problem associated with an and/or graph $G$ consists of finding a minimum weight solution subgraph $H$ of $G$, where the weight of a solution subgraph is the sum of the weights of its edges. Because of its wide applicability, in this work we develop a multivariate investigation of this optimization problem. In a previous paper [U. dos Santos Souza et al., J. Comput. Syst. Sci. 79, No. 7, 1156–1163 (2013; Zbl 1311.68078)] we have analyzed the complexity of such a problem under various aspects, including parameterized versions of it. However, the main open question has remained open: Is the problem of finding a solution subgraph of weight at most $k$ (where $k$ is the parameter) in FPT?

In this paper we answer negatively to this question, proving the W[1]-hardness of the problem, and its W[P]-completeness when zero-weight edges are allowed. We also show that the problem is fixed-parameter tractable when parameterized by the tree-width, but it is W[2]-hard with respect to the clique-width and $k$ as aggregated parameters. In addition, we show that when the out-edges of each or-vertex have all the same weight (a condition very common in practice), the problem becomes fixed-parameter tractable by the clique-width. Finally, using a framework developed by H. L. Bodlaender et al. [J. Comput. Syst. Sci. 75, No. 8, 423–434 (2009; Zbl 1192.68288)] and L. Fortnow and R. Santhanam [J. Comput. Syst. Sci. 77, No. 1, 91–106 (2011; Zbl 1233.68144)], based upon the notion of compositionality, we show that the tractable cases do not admit a polynomial kernel unless $NP \subseteq \text{coNP/poly}$, even restricted to instances without or-vertices with out-degree greater than two.

MSC:
05C99 Graph theory
68R10 Graph theory (including graph drawing) in computer science
68P05 Data structures
68Q15 Complexity classes (hierarchies, relations among complexity classes, etc.)

Keywords:
and/or graphs; W[1]-complete; W[1]-hard; FPT; tree-width; clique-width

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References:


[16] Diestel, R., Graph Theory, (2005), Springer


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