Coskun, Izzet; Huizenga, Jack; Woolf, Matthew

The effective cone of the moduli space of sheaves on the plane. (English) Zbl 1373.14042


This paper gives a detailed description of the effective cone of the Gieseker moduli space \( M(\xi) \) of stable coherent sheaves on \( \mathbb{P}^2 \) with Chern character \( \xi \).

Determination of the effective cone is reduced to the higher rank interpolation problem (see Problem 1.2), which asks to determine the minimal slope \( \mu^+(Q) \) satisfying some cohomological condition for a given sheaf \( U \in M(\xi) \). This interpolation problem has an algorithmic solution indicated in the main Theorem 1.3. The algorithm involves some graphs in the slope-discriminant plane \( \{ (\mu, \Delta) \} \), in particular the parabola \( Q_\xi \) encoding sheaves cohomologically orthogonal to \( U \in M(\xi) \). Theorem 1.3 is proved by a resolution of \( U \) in terms of a triplet of exceptional bundles, which comes from Beilinson spectral sequence.

The \( (\mu, \Delta) \)-plane and the graphs appearing in the interpolation problem enjoys remarkable arithmetic properties as discussed in §4. There is a curve \( \delta(\mu) \) giving the positive dimension criteria of the moduli space of vector bundles with slope \( \mu \). \( \delta(\mu) \) is fractal-like in the sense that it intersects with the line \( \Delta = 1/2 \) in a Cantor set \( C \). It is shown that the parabola \( Q_\xi \) does not intersect the line \( \Delta = 1/2 \) along \( C \).

The effective cone has two extremal edges. One edge is the pullback of the ample generator of Kronecker modules as discussed in §6. The other edge is the following description. For \( \xi \) with rank greater than 2, the Serre duality gives an isomorphism of Picard groups of moduli spaces preserving effective cones. Lower rank cases have more explicit description. For example, if the rank is 2, then the morphism \([J. Li, J. Differ. Geom. 37, No. 2, 417–466 (1993; Zbl 0809.14006)] M(\xi) \to M^{DUY}(\xi) \) to the Donaldson-Uhlenbeck-Yau moduli space determines the nef divisor giving the other edge.

Reviewer: Shintaro Yanagida (Nagoya)

MSC:

14J60 Vector bundles on surfaces and higher-dimensional varieties, and their moduli
14E30 Minimal model program (Mori theory, extremal rays)
14D20 Algebraic moduli problems, moduli of vector bundles
13D02 Syzygies, resolutions, complexes and commutative rings

Keywords:

moduli spaces of sheaves; Bridgeland stability; effective cone; Brill-Noether divisors

Full Text: DOI arXiv

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.