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Well-posedness for the Navier-Stokes equations with datum in the Sobolev spaces. (English)

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Summary: In this paper, we study local well-posedness for the Navier-Stokes equations with arbitrary initial data in homogeneous Sobolev spaces $\dot{H}_p^s(\mathbb{R}^d)$ for $d \geq 2$, $p > \frac{d}{2}$, and $\frac{d}{p} - 1 \leq s < \frac{d}{2p}$. The obtained result improves the known ones for $p > d$ and $s = 0$ (see [*M. Cannone*, *Ondelettes, paraproducts, et Navier-Stokes*. Paris: Diderot (1995; Zbl 1049.35517); *M. Cannone* and *Y. Meyer*, *Methods Appl. Anal.* 2, No. 3, 307–319 (1995; Zbl 0842.35074)]). In the case of critical indexes $s = \frac{d}{p} - 1$, we prove global well-posedness for Navier-Stokes equations when the norm of the initial value is small enough. This result is a generalization of the one in [*M. Cannone*, *Rev. Mat. Iberoam.* 13, No. 3, 515–541 (1997; Zbl 0897.35061)] in which $p = d$ and $s = 0$.

MSC:

35Q30 Navier-Stokes equations

76D05 Navier-Stokes equations for incompressible viscous fluids

35A01 Existence problems for PDEs: global existence, local existence, non-existence

35A02 Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness

Cited in 1 Document

Keywords:

Navier-Stokes equations; existence and uniqueness of local and global mild solutions; critical Sobolev and Besov spaces

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