Summary: This study examines necessary and sufficient conditions for a planar quintic Bézier curve to be a Pythagorean-hodograph (PH) curve. Quintic PH curves can be categorized into two classes according to the representation of their derivatives. While the first class has been studied by Farouki (1994) to be a family of regular curves already, a more succinct proof by introducing auxiliary control points is provided in this paper. Geometric characteristics of the second class of quintic PH curves are also studied. The key technique to simplify the discussion is to represent a planar Bézier curve with a complex polynomial in Bernstein form. Benefiting from such complex expression, algebraic characteristics of quintic PH curves can be described by nonlinear complex systems with respect to control points. By treating these systems with geometric methods, conditions for a quintic planar curve to be a PH curve can be described in terms of geometric constraints on its control polygon. Furthermore, we provide methods for the construction of the second class of quintic PH curves. In particular, parameter values of cusps can be explicitly determined in advance for irregular curves.

MSC:
53A04 Curves in Euclidean and related spaces
41A15 Spline approximation

Keywords:
geometric characteristic; Bézier curve; Pythagorean-hodograph; control polygon; Bézier curve

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References:
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