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Solving rank-constrained semidefinite programs in exact arithmetic. (English) Zbl 1380.90212

Summary: We consider the problem of minimizing a linear function over an affine section of the cone of positive semidefinite matrices, with the additional constraint that the feasible matrix has prescribed rank. When the rank constraint is active, this is a non-convex optimization problem, otherwise it is a semidefinite program. Both find numerous applications especially in systems control theory and combinatorial optimization, but even in more general contexts such as polynomial optimization or real algebra. While numerical algorithms exist for solving this problem, such as interior-point or Newton-like algorithms, in this paper we propose an approach based on symbolic computation. We design an exact algorithm for solving rank-constrained semidefinite programs, whose complexity is essentially quadratic on natural degree bounds associated to the given optimization problem: for subfamilies of the problem where the size of the feasible matrix, or the dimension of the affine section, is fixed, the algorithm is polynomial time. The algorithm works under assumptions on the input data: we prove that these assumptions are generically satisfied. We implement it in Maple and discuss practical experiments.

MSC:
90C22 Semidefinite programming

Keywords:
semidefinite programming; determinantal varieties; linear matrix inequalities; rank constraints; exact algorithms; computer algebra; polynomial optimization; spectrahedra; sums of squares

Software:
LMI\text{Rank}; \text{FGb}; \text{SPECTRA}; Maple; ISOLATE

Full Text: DOI

References: