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Symmetry reduction of the three-body problem based on Euler angles. (English)

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Summary: We consider the classical three-body problem with an arbitrary pair potential which depends on the inter-body distance. A general three-body configuration is set by three "radial" and three angular variables which determine the shape and orientation, respectively, of a triangle with the three bodies located at the vertices. The radial variables are given by the distances between a reference body and the other two, and by the angle at the reference body between the other two. Such radial variables set the potential energy of the system, and they are reminiscent of the inter-body distance in the two-body problem. On the other hand, the angular variables are the Euler angles relative to a rigid rotation of the triangle, and they are analogous to the polar and azimuthal angles of the vector between the two bodies in the two-body problem. We show that the rotational symmetry allows us to obtain a closed set of eight Hamilton equations of motion, whose generalized coordinates are the three radial variables and one additional angle, for which we provide the following geometrical interpretation. We consider the plane through a reference body, which is orthogonal to the line between the reference and the second body. We show that the angular variable above is the angle between the plane projection of the angular-momentum vector and the projection of the radius between the reference and the third body.

MSC:

70F07 Three-body problems
70G65 Symmetries, Lie group and Lie algebra methods for problems in mechanics

Full Text: DOI

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