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On some results of M. A. Malik concerning polynomials. (English) Zbl 1388.30003

Funct. Approximatio, Comment. Math. 57, No. 2, 143-149 (2017).

Summary: If $P(z)$ is a polynomial of degree n having all its zeros in $|z| \leq k, k \leqslant 1$, *N. A. Rather* et al. [“Inequalities involving the integrals of polynomials and their polar derivatives”, J. Class. Anal. 8, No. 1, 59–64 (2016; doi:10.7153/jca-08-05)] proved that for every $\alpha \in \mathbb{C}$ with $|\alpha| \geqslant k$ and $\gamma > 0$,

$$n(|\alpha| - k)\left\{ \int_0^{2\pi} \left| \frac{P(e^{i\theta})}{D_\alpha P(e^{i\theta})} \right|^\gamma d\theta \right\}^{\frac{1}{\gamma}} \leqslant \left\{ \int_0^{2\pi} \left| 1 + ke^{i\theta} \right|^\gamma d\theta \right\}^{\frac{1}{\gamma}}.$$

In this paper, we shall obtain a result which generalizes and sharpens the above inequality by obtaining a bound that depends upon the location of all the zeros of $P(z)$ rather than just on the location of the zero of largest modulus.

MSC:

30C10 Polynomials and rational functions of one complex variable
30A10 Inequalities in the complex plane
30C15 Zeros of polynomials, rational functions, and other analytic functions of one complex variable (e.g., zeros of functions with bounded Dirichlet integral)

Keywords:

polar derivative; integral mean estimate

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