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An all-speed Roe-type scheme and its asymptotic analysis of low Mach number behaviour.

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Summary: A new scheme, All-Speed-Roe scheme, was proposed for all speed flows. Compared with traditional preconditioned Roe scheme, All-Speed-Roe scheme changes non-linear eigenvalues in the numerical dissipation terms of Roe-type schemes. With an asymptotic analysis, the low Mach number behaviour of the scheme is studied theoretically in two ways. In one way, All-Speed-Roe scheme is regarded as finite magnification of Low-Speed-Roe scheme in the low Mach number limit. In the other way, a general form of All-Speed-Roe scheme is analyzed. Both ways demonstrate that All-Speed-Roe scheme has the same low Mach number behaviour as the original governing equation in the continuous case, which includes three important features: pressure variation scales with the square of the Mach number, the zero order velocity field is subject to a divergence constraint, and the second order pressure satisfies a Poisson-type equation in the case of constant-entropy. The analysis also leads to an unexpected conclusion that the velocity filed computed by traditional preconditioned Roe scheme does not satisfy the divergence constraint as the Mach number vanishes. Moreover, the analysis explains the reason of checkerboard decoupling and shows that momentum interpolation method provides a similar mechanism as traditional preconditioned Roe scheme inherently possesses to suppress checkerboard decoupling. In the end, general rulers for modifying non-linear eigenvalues are obtained. Finally, several numerical experiments are provided to support the theoretical analysis. All-Speed-Roe scheme has a sound foundation and is expected to be widely studied and applied to all speed flow calculations.

MSC:

76M20 Finite difference methods applied to problems in fluid mechanics

Keywords:

all-speed-Roe scheme; asymptotic analysis; shock-capturing scheme; low Mach number; checkerboard decoupling

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