Weissman, Martin H.

An illustrated theory of numbers. (English) Zbl 1390.11002


The book under review is an unusual introductory textbook on number theory. It contains almost 500 illustration which accompany the text addressing the basic elementary number theory. The book covers also some unusual topics not to be often found in standard textbooks on elementary number theory, as the class number, Ford circles, Conway’s topograph, Zolotarev lemma or Markov spectrum. It gives a “broadband” introduction to number theory and requires no special knowledge beyond standard high school mathematics and so it can serve as a very good readable introductory text. Beside this it can also be used as a good source for inspiration for those seeking “practical” motivations for an attractive introductory course. Numerous worked examples and exercises appropriately complement and extend the exposition. The pictures and comments on side margins give additional very suitably selected facts, comments, bibliographical and historical notes which spread throughout the text complete the general picture. The author uses a very good readable style. Last but not least, the US letter size of the book gives its content a dignified frame. All together, for its vivid exposition, carefully selected material and beautiful layout the book can be warmly recommended not only to students of all levels but also to teachers and professional mathematicians.

The introductory zeroth chapter shows how the “pebble technique” and its less known relatives can be used to deduced various summation identities. The first chapter centers around Euclidean algorithm and linear Diophantine equations in two unknowns and corresponding properties of the LCM and GCD. The second chapter deals with prime factorization. Besides known elementary results on prime factorization, the reader finds here several results which proofs are beyond the scope of the book, as the Green-Tao theorem, the Riemann hypotheses and its prime error-error consequence, the twin and Goldbach conjectures and their known best “approximations” as Zhang-Maynard bounded gaps or Chen theorems. Chapter 4 entitled “Rational and constructible numbers” contains besides standard results à la “median fractions lie between” and “integer generate all reduced fraction via medians”, Euclid’s and Descartes’ geometric constructions of some arithmetic quantities (e.g., square root) or that kissing fractions have tangent Ford cicles, or the proof of Dirichlet approximation theorem via Ford circles, etc. The reader finds here also comments on Fermat’s last theorem, the existence of Shimura and Taniyama conjecture, or the Thue-Siegel-Roth Theorem. Chapter 4 “Gaussian and Eisenstein integers” gives the reader a first touch with algebraic number theory in terms of prime decomposition, lifting of primes, ramifying or Chebyshev’s bias. Not to dip into further details, the second part “Modular arithmetic” having four chapters culminates in quadratic residues. The chapters of the third part “Quadratic forms” bear the titles the hodograph, definite forms and indefinite forms. In the last mentioned chapter the reader encounters Conway’s water, lakes, rivers or riverbends, and learns how to use this machinery to prove the finiteness of class number for indefinite forms and related topics as Markov spectrum, etc. Really a reading which can only be highly recommended.

Reviewer: Štefan Porubský (Praha)

MSC:

11–01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory
11Axx Elementary number theory
11Bxx Sequences and sets
11D04 Linear Diophantine equations
11E16 General binary quadratic forms
11N05 Distribution of primes
00A35 Methodology of mathematics

Keywords:

Euclidean algorithm; prime factorization; constructible numbers; Gaussian numbers; Eisenstein numbers;
modular arithmetic; modular words; modular dynamics; quadratic residues; quadratic forms; topograph; definite forms; indefinite forms