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Resonance tongues in the linear Sitnikov equation. (English) Zbl 1391.70032

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Summary: In this paper, we deal with a Hill's equation, depending on two parameters $e \in [0,1)$ and $\Lambda > 0$, that has applications to some problems in Celestial Mechanics of the Sitnikov type. Due to the nonlinearity of the eccentricity parameter e and the coexistence problem, the stability diagram in the (e,Λ)-plane presents unusual resonance tongues emerging from points $(0,(n/2)^2)$, $n = 1,2,\dots$ The tongues bounded by curves of eigenvalues corresponding to 2π-periodic solutions collapse into a single curve of coexistence (for which there exist two independent 2π-periodic eigenfunctions), whereas the remaining tongues have no pockets and are very thin. Unlike most of the literature related to resonance tongues and Sitnikov-type problems, the study of the tongues is made from a global point of view in the whole range of $e \in [0,1)$. Indeed, an interesting behavior of the tongues is found: almost all of them concentrate in a small Λ-interval $[1,9/8]$ as $e \to 1^-$. We apply the stability diagram of our equation to determine the regions for which the equilibrium of a Sitnikov $(N+1)$-body problem is stable in the sense of Lyapunov and the regions having symmetric periodic solutions with a given number of zeros. We also study the Lyapunov stability of the equilibrium in the center of mass of a curved Sitnikov problem.

MSC:

70F10 n-body problems
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Sitnikov problem; Hill's equation; resonance tongues; N-body problem

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