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**Random sampling of bandlimited signals on graphs.** (English) Zbl 1391.94367  
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**Summary:** We study the problem of sampling  $k$ -bandlimited signals on graphs. We propose two sampling strategies that consist in selecting a small subset of nodes at random. The first strategy is non-adaptive, i.e., independent of the graph structure, and its performance depends on a parameter called the graph coherence. On the contrary, the second strategy is adaptive but yields optimal results. Indeed, no more than  $O(k \log(k))$  measurements are sufficient to ensure an accurate and stable recovery of all  $k$ -bandlimited signals. This second strategy is based on a careful choice of the sampling distribution, which can be estimated quickly. Then, we propose a computationally efficient decoder to reconstruct  $k$ -bandlimited signals from their samples. We prove that it yields accurate reconstructions and that it is also stable to noise. Finally, we conduct several experiments to test these techniques.

**MSC:**

[94A12](#) Signal theory (characterization, reconstruction, filtering, etc.)  
[94A20](#) Sampling theory in information and communication theory  
[05C90](#) Applications of graph theory  
[94C15](#) Applications of graph theory to circuits and networks

Cited in **10** Documents

**Keywords:**

graph signal processing; sampling; compressed sensing

**Software:**

PyGSP; GSPBOX

**Full Text:** [DOI](#) [arXiv](#)

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