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$q$-log-convexity from linear transformations and polynomials with only real zeros.  (English)


Summary: In this paper, we mainly study the stability of iterated polynomials and linear transformations preserving the strong $q$-log-convexity of polynomials.

Let $[T_{n,k}]_{n,k \geq 0}$ be an array of nonnegative numbers. We give some criteria for the linear transformation

$$y_n(q) = \sum_{k=0}^{n} T_{n,k}x_k(q)$$


The stability property of iterated polynomials implies the $q$-log-convexity. By applying the method of interlacing of zeros, we also present two criteria for the stability of the iterated Sturm sequences and $q$-log-convexity of polynomials. As consequences, we get the stabilites of iterated Eulerian polynomials of types A and B, and their $q$-analogs. In addition, we also prove that the generating functions of alternating runs of types A and B, the longest alternating subsequence and up-down runs of permutations form a $q$-log-convex sequence, respectively.

MSC:

05A15 Exact enumeration problems, generating functions
05A05 Permutations, words, matrices
05A20 Combinatorial inequalities
12D10 Polynomials in real and complex fields: location of zeros (algebraic theorems)
11B83 Special sequences and polynomials

Software:

OEIS

Full Text: DOI arXiv

References:


Brenti, F., $q$-$\phi$-Eulerian polynomials arising from Coxeter groups, European J. Combin., 15, 417-441, (1994) · Zbl 0809.050012


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Zhao, A. F.Y., The combinatorics on permutations and derangements of type $SB$, (2011), Nankai University, Ph.D dissertation


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