Let $k$ be an algebraically closed field of characteristic $p \geq 0$ and $\varphi : S \to B$ be an elliptic surface of Kodaira dimension 1 over $k$. It is known that if $S$ is algebraic and $p \neq 2$, then for $m \geq 14$, $mK_S$ gives a structure of elliptic surface and 14 is the best possible. It is also known, when $p = 2$, for $m \geq 12$, $mK_S$ gives a structure of elliptic surface and 12 is the best possible. [T. Katsuro and K. Ueno, Math. Ann. 272, 291–330 (1985; Zbl 0553.14019)], [T. Katsura, Compos. Math. 97, No. 1–2, 119–134 (1995; Zbl 0860.14036)]. In the paper under review, the author proves that a similar result for quasi-elliptic surfaces over a field of characteristic 3. The main result of this paper is following:

**Theorem 3.1**. Assume that the characteristic $p = 3$. Then for any quasi-elliptic surface $f : S \to B$ with $\kappa(S) = 1$ over $k$ and for any $m \geq 5$, the multicanonical system $|mK_S|$ gives the unique structure of quasi-elliptic surface, and the number 5 is best possible.

The key step of the proof of the main result is the inequality $\chi(O_S) \geq (1 - g)/3$ (Lemma 2.2). Here $g$ is the genus of the base curve $B$.

For the entire collection see [Zbl 1382.14002].

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