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Deodhar elements in Kazhdan-Lusztig theory. (English. French summary) Zbl 1393.20001

Summary: The Kazhdan-Lusztig polynomials for finite Weyl groups arise in representation theory as well as the geometry of Schubert varieties. It was proved very soon after their introduction that they have nonnegative integer coefficients, but no simple all positive interpretation for them is known in general. Deodhar has given a framework, which generally involves recursion, to express the Kazhdan-Lusztig polynomials in a very attractive form.

We use a new kind of pattern-avoidance that can be defined for general Coxeter groups to characterize when Deodhar’s algorithm yields a non-recursive combinatorial formula for Kazhdan-Lusztig polynomials $P_{x,w}(q)$ of finite Weyl groups. This generalizes results of Billey-Warrington which identified the 321-hexagon-avoiding permutations, and Fan-Green which identified the fully-tight Coxeter groups. We also show that the leading coefficient known as $\mu(x, w)$ for these Kazhdan-Lusztig polynomials is always either 0 or 1. Finally, we generalize the simple combinatorial formula for the Kazhdan-Lusztig polynomials of the 321-hexagon-avoiding permutations to the case when $w$ is hexagon avoiding and maximally clustered.

For the entire collection see [Zbl 1173.05001].

MSC:

20C08 Hecke algebras and their representations
20F55 Reflection and Coxeter groups (group-theoretic aspects)
05E15 Combinatorial aspects of groups and algebras (MSC2010)

Keywords:
Kazhdan-Lusztig polynomial; pattern avoidance; 321-hexagon; freely-braided permutation; maximally-clustered permutation; 2-sided weak Bruhat order

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