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Togliatti systems and Galois coverings. (English) Zbl 1395.13019


A Togliatti system is an Artinian ideal \( I \) generated by forms of degree \( d \) that define a morphism \( \varphi : \mathbb{P}^n \to \mathbb{P}^{r-1} \) whose image \( X \) satisfies at least one Laplace equation of order \( d - 1 \).

In this article, the authors study Togliatti systems with the additional property that \( \varphi \) is a Galois covering with cyclic Galois group \( \mathbb{Z}/d\mathbb{Z} \) (“GT systems”). They completely classify GT systems if \( d \) is a prime power. For general \( d \), a complete classification under an additional symmetry assumption on the generating monomials is given. This constitutes essential progress over existing results for \( d = 3 \).

The ideal of the rational surface associated to a GT system is generated by quadrics and, if \( d \) is odd, by cubics. It has three singular points whose respective types are determined. Finally, a relation between minimal monomial GT systems and Ceva configurations of \( d^2 \) lines and \( 3d \) points such that each point is incident with \( d \) lines and each line contains 3 points, is unveiled.

Reviewer: Hans-Peter Schröcker (Innsbruck)

MSC:

13E10 Commutative Artinian rings and modules, finite-dimensional algebras
14M25 Toric varieties, Newton polyhedra, Okounkov bodies
14N05 Projective techniques in algebraic geometry
14N15 Classical problems, Schubert calculus
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References:

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