Given a metric space $M$ which is proper (that is balls in $M$ are compact) there is a nice compact topology on the set $\mathcal{C}(M)$ of its closed subsets which makes a compact space of $\mathcal{C}(M)$. It has been used extensively in group theory, starting with the paper of C. Chabauty [Bull. Soc. Math. Fr. 78, 143–151 (1950; Zbl 0039.04101)] where it was introduced.

The Chabauty topology on $\mathcal{C}(M)$ is usually defined by describing a basis of open subsets (given by those subsets of $\mathcal{C}(M)$ comprising closed sets which avoid – respectively intersect – a given compact – respectively open – subset of $M$) but it is well-known that it is metrisable. In this short note the author describes a very explicit way of constructing a distance which induces the Chabauty topology; namely he proves that integrating Hausdorff distances in an exhaustion by concentric balls against a sufficiently decreasing function does the job.

Reviewer: Jean Raimbault (Toulouse)