Webb, Gary
Magnetohydrodynamics and fluid dynamics: action principles and conservation laws. (English) [Zbl 1397.76001]

The book focuses on conservation laws in magnetohydrodynamics and gas dynamics. The conservation laws are essential in fusion and space plasmas as well as in geophysical fluid dynamics; they should be used to understand fluid vorticity or to investigate the stability of steady flows. The book discusses several variants of helicity including kinetic helicity, cross helicity and magnetic helicity. The author develops the non-canonical Hamiltonian approach to MHD using the non-canonical Poisson bracket. He also presents Anco and Bluman’s direct methods for deriving conservation laws. The ten Galilean-Lie point symmetries of the fundamental action give rise to the conservation of energy, momentum, angular momentum and center of mass via Noether’s first theorem. A range of examples illustrates topological invariants in MHD and fluid dynamics, including the Hopf invariant, the Calugareanu invariant, the MHD topological solitons and the Parker-Archimedes spiral magnetic field. The Aharonov-Bohm interpretation of magnetic and cross helicity is discussed. The Lagrangian map is used to obtain a class of solutions for incompressible MHD. In closing, examples of magnetosonic $N$-waves illustrate the wave number and group velocity concepts for MHD waves. This book could be used as a guide to the applications of conservation laws in MHD.

Reviewer: Titus Petrila (Cluj-Napoca)

MSC:
76-02 Research exposition (monographs, survey articles) pertaining to fluid mechanics
76W05 Magnetohydrodynamics and electrohydrodynamics
76M60 Symmetry analysis, Lie group and Lie algebra methods applied to problems in fluid mechanics
35L65 Hyperbolic conservation laws
00A79 Physics

Keywords:
topological invariant; kinetic helicity; cross helicity; magnetic helicity; non-canonical Poisson bracket; point symmetries; Noether’s theorem; Lagrangian map

Full Text: DOI