Meyerovitch, Tom; Perl, Idan; Tointon, Matthew; Yadin, Ariel
Polynomials and harmonic functions on discrete groups. (English) [Zbl 1401.43001]

finitely generated virtually nilpotent group, the restriction of a harmonic function of polynomial growth
to a torsion-free nilpotent subgroup of finite index is always a polynomial in the Mal’cev coordinates of
that subgroup. For general groups, vanishing of higher-order discrete derivatives gives a natural notion
of polynomial maps, which has been considered by A. Leibman [Isr. J. Math. 129, 29–60 (2002; Zbl
1007.20035)] and others. We provide a simple proof of Alexopoulos’s result [loc. cit.] using this notion of
polynomials under the weaker hypothesis that the space of harmonic functions of polynomial growth
of degree at most \(k\) is finite-dimensional. We also prove that for a finitely generated group the Laplacian
maps the polynomials of degree \(k\) surjectively onto the polynomials of degree \(k - 2\). We then present
some corollaries. In particular, we calculate precisely the dimension of the space of harmonic functions
of polynomial growth of degree at most \(k\) on a virtually nilpotent group, extending an old result of H. A.
a more recent result of B. Hua and J. Jost [Math. Z. 280, No. 1–2, 551–567 (2015; Zbl 1369.31014)].

MSC:
43A05 Measures on groups and semigroups, etc.
20F65 Geometric group theory
20F18 Nilpotent groups
20F19 Generalizations of solvable and nilpotent groups
60B15 Probability measures on groups or semigroups, Fourier transforms, factorization

Full Text: DOI arXiv

References:
(1972) · Zbl 0259.20045
[3] Benjamini, Itai; Duminil-Copin, Hugo; Kozma, Gady; Yadin, Ariel, Disorder, entropy and harmonic functions, Ann. Probab.,
Zbl 0928.53030 · doi:10.2307/2952459
(2012) · Zbl 1251.37012 · doi:10.4007/annals.2012.175.2.2
Zbl 0473.20035
Zbl 0294.43003
Zbl 1246.20038 · doi:10.1090/S0894-0347-09-00658-4


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