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Symmetries and multipeakon solutions for the modified two-component Camassa-Holm system.

Summary: Compared with the two-component Camassa-Holm system, the modified two-component Camassa-Holm system introduces a regularized density which makes possible the existence of solutions of lower regularity, and in particular of multipeakon solutions. In this paper, we derive a new pointwise invariant for the modified two-component Camassa-Holm system. The derivation of the invariant uses directly the symmetry of the system, following the classical argument of Noether’s theorem. The existence of the multipeakon solutions can be directly inferred from this pointwise invariant. This derivation shows the strong connection between symmetries and the existence of special solutions. The observation also holds for the scalar Camassa-Holm equation and, for comparison, we have also included the corresponding derivation. Finally, we compute explicitly the solutions obtained for the peakon-antipeakon case. We observe the existence of a periodic solution which has not been reported in the literature previously. This case shows the attractive effect that the introduction of an elastic potential can have on the solutions.

For the entire collection see [Zbl 1390.35006].

MSC:
35B06 Symmetries, invariants, etc. in context of PDEs
35B10 Periodic solutions to PDEs
35F55 Initial value problems for systems of nonlinear first-order PDEs

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