Summary: We classify instances of quantum pseudo-telepathy in the graph isomorphism game, exploiting the recently discovered connection between quantum information and the theory of quantum automorphism groups. Specifically, we show that graphs quantum isomorphic to a given graph are in bijective correspondence with Morita equivalence classes of certain Frobenius algebras in the category of finite-dimensional representations of the quantum automorphism algebra of that graph. We show that such a Frobenius algebra may be constructed from a central type subgroup of the classical automorphism group, whose action on the graph has coisotropic vertex stabilisers. In particular, if the original graph has no quantum symmetries, quantum isomorphic graphs are classified by such subgroups. We show that all quantum isomorphic graph pairs corresponding to a well-known family of binary constraint systems arise from this group-theoretical construction. We use our classification to show that, of the small order vertex-transitive graphs with no quantum symmetry, none is quantum isomorphic to a non-isomorphic graph. We show that this is in fact asymptotically almost surely true of all graphs.

MSC:
05C60 Isomorphism problems in graph theory (reconstruction conjecture, etc.) and homomorphisms (subgraph embedding, etc.)
05C57 Games on graphs (graph-theoretic aspects)
05C15 Coloring of graphs and hypergraphs
91A43 Games involving graphs
17B37 Quantum groups (quantized enveloping algebras) and related deformations
81P99 Foundations, quantum information and its processing, quantum axioms, and philosophy

Keywords:
quantum automorphism groups of graphs; quantum isomorphic graph

Full Text: DOI

References:

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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