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Tiered trees, weights, and \(q\)-Eulerian numbers.  (English)  [Zbl 1407.05048]  

Summary: Maxmin trees are labeled trees with the property that each vertex is either a local maximum or a local minimum. Such trees were originally introduced by A. Postnikov [J. Comb. Theory, Ser. A 79, No. 2, 369-366 (1997; Zbl 0876.05042)], who gave a formula to count them and different combinatorial interpretations for their number. In this paper we generalize this construction and define tiered trees by allowing more than two classes of vertices. Tiered trees arise naturally when counting the absolutely indecomposable representations of certain quivers, and also when one enumerates torus orbits on certain homogeneous varieties. We define a notion of weight for tiered trees and prove bijections between various weight 0 tiered trees and other combinatorial objects; in particular order \(n\) weight 0 maxmin trees are naturally in bijection with permutations on \(n-1\) letters. We conclude by using our weight function to define a new \(q\)-analogue of the Eulerian numbers.

MSC:  
05C05 Trees  
05C35 Extremal problems in graph theory  
05A05 Permutations, words, matrices  
11B68 Bernoulli and Euler numbers and polynomials

Keywords:  
maxmin trees; intransitive trees; Eulerian numbers; \(q\)-Eulerian numbers; nonambiguous trees; permutations

Software:  
OEIS

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References:

Sloane, N. J.A., Online encyclopedia of integer sequences, available at - Zbl 1044.11108


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