This paper is devoted to the study of a 19-dimensional family $\mathcal{U}$ of irreducible holomorphic symplectic (IHS) fourfolds, which are deformation equivalent to a Hilbert square on a $K3$ surface and with polarization of degree 4 with respect to the Beauville-Bogomolov-Fujiki form. In particular, the authors describe three geometric constructions giving rise to elements in $\mathcal{U}$.

The first description, explained in Section 3, is as double covers of some Lagrangian degeneracy loci inside a cone over $\mathbb{P}^2 \times \mathbb{P}^2$. More precisely, given two 3-dimensional vector spaces $U_1$ and $U_2$ with fixed volume forms, consider the cone $C_{U_1} := C(\mathbb{P}(\mathbb{A}^2 U_1) \times \mathbb{P}(U_2)) \subset \mathbb{P}^2$ over the Segre embedding of $\mathbb{P}(\mathbb{A}^2 U_1) \times \mathbb{P}(U_2)$. For every general Lagrangian subspace $A$ of $\mathbb{A}^2 U_1 \oplus U_2 \oplus U_1 \otimes \mathbb{A}^2 U_2$, there is an associated 4-dimensional quartic section $D_A^4$ of $C_{U_1}$, called EPW quartic section. Then the double cover $X_A$ of $D_A^4$ branched along its singular locus is a IHS fourfold in the family $\mathcal{U}$ (Theorem 1.1).

Secondly, in Section 4 they consider the Hilbert scheme of conics on Verra fourfolds. Recall that a Verra fourfold is a smooth intersection of the cone $\mathbb{A}^2 U_1 \oplus U_2 \oplus U_1 \otimes \mathbb{A}^2 U_2$, there is an associated 4-dimensional quartic section $D_A^4$ of $C_{U_1}$, called EPW quartic section. Then the double cover $X_A$ of $D_A^4$ branched along its singular locus is a IHS fourfold in the family $\mathcal{U}$ (Theorem 1.1).

The third construction is explained in Section 5. The authors prove that a general element in $\mathcal{U}$ is isomorphic to a moduli space of twisted stable sheaves on a degree-2 polarized $K3$ surface with Brauer class of order 2 (Theorem 1.3). This result completes the geometric realization of IHS fourfolds arising from 2-torsion elements in the Brauer group of a degree-2 polarized $K3$ surface, classified by B. van Geemen [Adv. Math. 197, 222–247 (2005; Zbl 1082.14040)].

The description of Theorem 1.1 allows also to complete the classification of geometric realizations of anti-symplectic involutions on IHS fourfolds of type $K3^{[2]}$. Indeed, the family $\mathcal{U}$ is the unique 19-dimensional irreducible family of IHS fourfolds of type $K3^{[2]}$ that is not in the closure of the family of double EPW sextics, such that each element admits an antisymplectic involution (Corollary 1.5). Section 2 is devoted to the description of the first two constructions in the preliminary case of Kummer surfaces. Finally, Section 6 is devoted to the computation of some invariants of the 2-dimensional fixed loci of the involution on the elements of $\mathcal{U}$.

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MSC:

14D06 Fibrations, degenerations in algebraic geometry
14D20 Algebraic moduli problems, moduli of vector bundles
14E25 Embeddings in algebraic geometry
14J32 Calabi-Yau manifolds (algebrao-geometric aspects)
14J35 4-folds

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