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Binary constraint satisfaction problems defined by excluded topological minors. (English)

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Summary: The binary Constraint Satisfaction Problem (CSP) is to decide whether there exists an assignment to a set of variables which satisfies specified constraints between pairs of variables. A binary CSP instance can be presented as a labelled graph encoding both the forms of the constraints and where they are imposed. We consider subproblems defined by restricting the allowed form of this graph. One type of restriction is to forbid certain specified substructures (patterns). This captures some tractable classes of the CSP, but does not capture classes defined by language restrictions, or the well-known structural property of acyclicity.

We extend the notion of pattern and introduce the notion of a topological minor of a binary CSP instance. By forbidding a *finite* set of patterns from occurring as topological minors we obtain a compact mechanism for expressing novel tractable subproblems of the CSP, including new generalisations of the class of acyclic instances.

MSC:

- [68T20](#) Problem solving in the context of artificial intelligence (heuristics, search strategies, etc.) Cited in 2 Documents
- [05C83](#) Graph minors
- [68Q25](#) Analysis of algorithms and problem complexity

Keywords:

[constraint satisfaction](#); [forbidden substructures](#); [forbidden patterns](#); [topological minors](#)

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