Das, Kinkar Ch.; Liu, Muhuo

On two conjectures of spectral graph theory. (English) [Zbl 1409.05128]

Summary: Let $G = (V, E)$ be a simple graph. Denote by $D(G)$ the diagonal matrix of its vertex degrees and by $A(G)$ its adjacency matrix. Then the Laplacian matrix and the signless Laplacian matrix of $G$ are $L(G) = D(G) - A(G)$ and $Q(G) = D(G) + A(G)$, respectively. Also denote by $\lambda_1(G)$, $a(G)$, $q_1(G)$ and $\delta(G)$ the largest eigenvalue of $A(G)$, the second smallest eigenvalue of $L(G)$, the largest eigenvalue of $Q(G)$ and the minimum degree of $G$, respectively. In this paper, we give partial proofs to the following two conjectures:

(i) M. Aouchiche [Comparaison automatisée d’invariants en théorie des graphes. Montréal: École Polytechnique de Montréal (PhD Thesis) (2006)] if $G$ is a connected graph, then $a(G)/\delta(G)$ is minimum for graph composed of 2 triangles linked with a path.

(ii) M. Aouchiche and P. Hansen [Linear Algebra Appl. 432, No. 9, 2293-2322 (2010; Zbl 1218.05087)] and D. Cvetković et al. [Publ. Inst. Math., Nouv. Sér. 81(95), 11-27 (2007; Zbl 1164.05038)] if $G$ is a connected graph with $n \geq 4$ vertices, then $q_1(G) - 2\lambda_1(G) \leq n - 2\sqrt{n-1}$ with equality holding if and only if $G \cong K_{1,n-1}$.

MSC:
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)
15A18 Eigenvalues, singular values, and eigenvectors
15B36 Matrices of integers

Keywords:
graph; spectral radius; signless Laplacian spectral radius; algebraic connectivity

Software:
SageMath

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References:


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