Summary: In this paper, we characterize the hyperbolic product graphs for the Cartesian sum $G_1 \oplus G_2$: $G_1 \oplus G_2$ is always hyperbolic, unless either $G_1$ or $G_2$ is the trivial graph (the graph with a single vertex); if $G_1$ or $G_2$ is the trivial graph, then $G_1 \oplus G_2$ is hyperbolic if and only if $G_2$ or $G_1$ is hyperbolic, respectively. Besides, we characterize the Cartesian sums with hyperbolicity constant $\delta(G_1 \oplus G_2) = t$ for every value of $t$. Furthermore, we obtain the sharp inequalities $1 \leq \delta(G_1 \oplus G_2) \leq 3/2$ for every non-trivial graphs $G_1, G_2$. In addition, we obtain simple formulas for the hyperbolicity constant of the Cartesian sum of many graphs. Finally, we prove the inequalities $3/2 \leq \delta(G_1 \oplus G_2) \leq 2$ for the complement of $G_1 \oplus G_2$ for every $G_1, G_2$ with min\{diam $V(G_1)$, diam $V(G_2)$\} $\geq 3$.

MSC:
05C75 Structural characterization of families of graphs
05C12 Distance in graphs
05A20 Combinatorial inequalities

Keywords:
Cartesian sum of graphs; geodesics; Gromov hyperbolicity; complement of graphs

References:


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