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Moduli spaces of compact Riemann surfaces: their complex structures and an overview of major results. (English) [\[Zbl 1409.32007\]](#)

Ji, Lizhen (ed.) et al., Uniformization, Riemann-Hilbert correspondence, Calabi-Yau manifolds and Picard-Fuchs equations. Based on the conference, Institute Mittag-Leffler, Stockholm, Sweden, July 13–18, 2015. Somerville, MA: International Press; Beijing: Higher Education Press. Adv. Lect. Math. (ALM) 42, 111-156 (2018).

Summary: In this paper, we describe the history of defining and understanding the *right complex structure* on the moduli space \mathcal{M}_g of compact Riemann surfaces of genus g and the associated Teichmüller space \mathcal{T}_g . In particular we discuss the motivation for the complex structure on \mathcal{T}_g constructed by Ahlfors and Bers through a result from the theory of variation of Hodge structures, and explain how the proper meaning of the module spaces and their right complex structures can be understood when moduli spaces are formulated in terms representability of moduli functors. Using the representability of the module functor of marked Riemann surfaces, we prove that other complex structures put on \mathcal{T}_g by different methods are isomorphic to the right one. In the end, we also mention some results in the development of these moduli spaces, most of which related to the complex structure on \mathcal{T}_g and \mathcal{M}_g .

For the entire collection see [\[Zbl 1398.14003\]](#).

MSC:

- [32G15](#) Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)
- [30F60](#) Teichmüller theory for Riemann surfaces
- [32-02](#) Research exposition (monographs, survey articles) pertaining to several complex variables and analytic spaces
- [30-02](#) Research exposition (monographs, survey articles) pertaining to functions of a complex variable

Keywords:

[compact Riemann surfaces](#); [moduli space](#); [Teichmüller space](#); [complex structures](#)