Schillewaert, Jeroen; van Maldeghem, Hendrik

On the varieties of the second row of the split Freudenthal-Tits magic square. (Sur les variétés de la deuxième rangée du carré magique deployé de Freudenthal-Tits.)


The split composition algebras of the second row of the Freudenthal-Tits magic square give rise to the complex Severi varieties, characterized by F. L. Zak [Tangents and secants of algebraic varieties. Providence, RI: American Mathematical Society (1993; Zbl 0795.14018)] as extremal and secant defective. Veronese varieties in some finite projective spaces have been characterized in terms of their conics by F. Mazzocca and N. Melone [Discrete Math. 48, 243–252 (1984; Zbl 0537.51014)]. Inspired by the latter characterization, the authors define so called Mazzocca-Melone sets which generalize the complex Severi varieties to arbitrary fields only requiring combinatorial properties. Their main result is a characterization of Mazzocca-Melone sets as “subvarieties of the varieties of the second row of the split Freudenthal-Tits Magic Square that are controlled by the diagram of the corresponding building”; a combinatorial analogue of the classification of complex Severi varieties is achieved as a corollary of this.

In more detail, let $\mathcal{P}$ be an $N$-dimensional projective space and $X$ a subset which generates $\mathcal{P}$, $\Xi$ a family of $(d+1)$-dimensional subspaces $\xi$, $d > 0$ which all intersect $X$ in a non-singular split quadric $X(\xi)$. The authors call $(X, \Xi)$ a Mazzocca-Melone set of split type $d$ iff (1) for all $x, y \in X$ exists $\xi \in \Xi: x, y \in \xi$; (2) for all $\xi_1, \xi_2 \in \Xi, \xi_1 \neq \xi_2: \xi_1 \cap \xi_2 \subset X$; (3) for all $x \in X$ the tangent space to $X(\xi)$ through $x$ is contained in a $2d$-dimensional space.

The authors prove that, up to projective equivalence, proper (i.e., $|\Xi| > 1$) Mazzocca-Melone sets $(X, \Xi)$ of split type $d > 0$ must be one of the following: a quadric Veronese variety $(d = 1; N = 5)$, Segre varieties $(d = 2; N = 5, 7, 8)$, line Grassmannians $(d = 4; N = 9, 14)$, a half-spin variety $(d = 6; N = 15)$ or an exceptional variety of type $E_6$ $(d = 8; N = 26)$.

If furthermore $N \geq 3d + 2$ or if there exists a $2d$-dimensional tangent space, then only a quadric Veronese variety, a Segre variety of the form $S_{2,2}$, a line Grassmannian of the form $G_{1,5}$ or an exceptional variety of type $E_6$ are possible, $N = 3d + 2$, and all tangent spaces have dimension $2d$.

The proof of these results for $d = 1, 2, 3$ relies on former work of the authors, see [Bull. Belg. Math. Soc. - Simon Stevin 20, No. 1, 19–25 (2013; Zbl 1271.51001); Adv. Math. 262, 784–822 (2014; Zbl 1383.51005); Glasgow Math. J. 58, No. 2, 293–311 (2016; Zbl 1354.51008)]. The treatment of the case $d = 4$ now completes the characterization of split varieties. Remarks and results on the non-split case are also given. Throughout, the authors explain their motivation and methods thoroughly.

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MSC:

51E24 Buildings and the geometry of diagrams
51A45 Incidence structures embeddable into projective geometries
51M35 Synthetic treatment of fundamental manifolds in projective geometries (Grassmannians, Veronesians and their generalizations)
14M12 Determinantal varieties
14M15 Grassmannians, Schubert varieties, flag manifolds
17C37 Associated geometries of Jordan algebras
20G15 Linear algebraic groups over arbitrary fields

Keywords:

Severi variety; Veronese variety; Segre variety; Grassmann variety; Tits-building; Freudenthal-Tits magic square; Freudenthal magic square; split composition algebra

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References:


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