Schumacher, Georg

Moduli of canonically polarized manifolds, higher order Kodaira-Spencer maps, and an analogy to Calabi-Yau manifolds. (English) Zbl 1411.32018

Ji, Lizhen (ed.) et al., Uniformization, Riemann-Hilbert correspondence, Calabi-Yau manifolds and Picard-Fuchs equations. Based on the conference, Institute Mittag-Leffler, Stockholm, Sweden, July 13–18, 2015. Somerville, MA: International Press; Beijing: Higher Education Press. Adv. Lect. Math. (ALM) 42, 369-399 (2018).

Summary: Yau's solution of the Calabi conjecture made a differential geometric study of moduli spaces possible. The Weil-Petersson metric, which is a Kähler metric for moduli of canonically polarized manifolds, and for polarized Calabi-Yau manifolds, reflects the variation of the Kähler-Einstein metrics in a holomorphic family. Incidentally the existence of Kähler-Einstein metrics implies an analytic proof for the existence of the corresponding moduli spaces. In order to show that the moduli stack of canonically polarized manifolds is hyperbolic, one has to consider higher order Kodaira-Spencer maps. We compute the curvature of the related twisted Hodge sheaves $R^{n-p}f_*\Omega_{\mathcal{X}/S}(\mathcal{K}_{\mathcal{X},S})$ for holomorphic families $f:\mathcal{X}\to S$. The result exhibits a formal analogy to the classical curvature formula for Hodge bundles for families of Calabi-Yau manifolds. We construct a Finsler metric of negative curvature on the moduli stack of canonically polarized manifolds, whose curvature is bounded from above by a negative constant. An extra argument together with Demailly's version of the Ahlfors Lemma are needed for those points, where the twisted Hodge sheaves are not locally free.

For the entire collection see [Zbl 1398.14003].

MSC:

32Q25 Calabi-Yau theory (complex-analytic aspects)
14J32 Calabi-Yau manifolds (algebro-geometric aspects)

Keywords:

Calabi-Yau manifolds; Kähler-Einstein metrics

Full Text: arXiv