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Stieltjes polynomials and related quadrature formulae for a class of weight functions. II.
(English) [Zbl 1411.41022]

Summary: Consider a (nonnegative) measure $d\sigma$ with support in the interval $[a, b]$ such that the respective orthogonal polynomials satisfy a three-term recurrence relation with coefficients

$$\alpha_n = \begin{cases} 
\alpha_e, & n \text{ even}, \\
\alpha_o, & n \text{ odd},
\end{cases} \quad \beta_n = \beta \text{ for } n \geq \ell,$$

where $\alpha_e, \alpha_o, \beta$ and $\ell$ are specific constants. We show that the corresponding Stieltjes polynomials, above the index $2\ell - 1$, have a very simple and useful representation in terms of the orthogonal polynomials. As a result of this, the Gauss-Kronrod quadrature formula for $d\sigma$ has all the desirable properties, namely, the interlacing of nodes, their inclusion in the closed interval $[a, b]$ (under an additional assumption on $d\sigma$), and the positivity of all weights, while the formula enjoys an elevated degree of exactness. Furthermore, the interpolatory quadrature formula based on the zeros of the Stieltjes polynomials has positive weights and also elevated degree of exactness. It turns out that this formula is the anti-Gaussian formula for $d\sigma$, while the resulting averaged Gaussian formula coincides with the Gauss-Kronrod formula for this measure. Moreover, we show that the only positive and even measure $d\sigma$ on $(-a, a)$ for which the Gauss-Kronrod formula is almost of Chebyshev type, i.e., it has almost all of its weights equal, is the measure $d\sigma(t) = (a^2 - t^2)^{-1/2}dt$.


MSC:
41A55 Approximate quadratures
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)
65D32 Numerical quadrature and cubature formulas

Full Text: DOI

References:


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