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**Global and local behavior of a class of  $\xi^{(s)}$ -QSO.** (English) Zbl 1412.37050

J. Nonlinear Sci. Appl. 10, No. 9, 4834-4845 (2017).

Summary: A quadratic stochastic operator (QSO) describes the time evolution of different species in biology. The main problem with regard to a nonlinear operator is to study its behavior. This has not been studied in depth; even QSOs, which are the simplest nonlinear operators, have not been studied thoroughly. This paper investigates the global behavior of an operator taken from  $\xi^{(s)}$ -QSO when the parameter  $a = \frac{1}{2}$ . Moreover, we study the local behavior of this operator at each value of  $a$ , where  $0 < a < 1$ .

#### MSC:

[37E99](#) Low-dimensional dynamical systems

[37N25](#) Dynamical systems in biology

[39B82](#) Stability, separation, extension, and related topics for functional equations

[47H60](#) Multilinear and polynomial operators

[92D25](#) Population dynamics (general)

#### Keywords:

quadratic stochastic operator; local behavior; global behavior

**Full Text:** [DOI](#)

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