Borodin, O. V.; Ivanova, A. O.
An improvement of Lebesgue’s description of edges in 3-polytopes and faces in plane quadrangulations. (English) Zbl 1414.05089

Summary: An edge $e$ in a 3-polytope is of type $(k_1, k_2, k_3, k_4)$ if the set of degrees of the vertices and faces incident with $e$ is majorized by the vector $(k_1, k_2, k_3, k_4)$.

H. Lebesgue [J. Math. Pures Appl. (9) 19, 27–43 (1940; Zbl 0024.28701)] proved that every 3-polytope has an edge of one of the types $(3, 3, 3, \infty)$, $(3, 3, 4, 11)$, $(3, 3, 5, 7)$, $(3, 4, 4, 5)$.

This also provides a description of the faces of quadrangulated 3-polytopes in terms of degrees of their incident vertices.

The purpose of our paper is to prove that every 3-polytope has an edge of one of the types $(3, 3, 3, \infty)$, $(3, 3, 4, 9)$, $(3, 3, 5, 6)$, $(3, 4, 4, 5)$, where all parameters except possibly 9 are best possible. We believe that 9 here is sharp and thus the whole description is tight.

Our proof relies on the discharging method.

MSC:
05C10 Planar graphs; geometric and topological aspects of graph theory
52B05 Combinatorial properties of polytopes and polyhedra (number of faces, shortest paths, etc.)

Keywords:
plane graph; 3-polytope; edge; structural properties; height; weight

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