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Three-dimensional bifurcation analysis of a predator-prey model with uncertain formulation. (English) [Zbl 1414.34031](#)
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This paper focuses on structural sensitivity in using alternative choices in model formulation. An example of a predator-prey system with an uncertain functional response that exhibits complex bifurcations is introduced to illustrate how qualitative changes in bifurcations occur during a small continuous change between two acceptable model formulations. A bifurcation analysis reveals that most changes that occur with the change in formulation are relevant to a codimension-three degenerated Bogdanov-Takens bifurcation. Its canonical unfolding is derived, and its analysis highlights the differences in the phase portraits predicted with the two model formulations. The concept and method in this paper are novel and general and can be applied to other fields involving the modeling of complex systems.

Reviewer: [Xinyu Song \(Xinyang\)](#)

MSC:

- [34C60](#) Qualitative investigation and simulation of ordinary differential equation models Cited in 1 Document
- [34C23](#) Bifurcation theory for ordinary differential equations
- [34D30](#) Structural stability and analogous concepts of solutions to ordinary differential equations
- [37G05](#) Normal forms for dynamical systems
- [92D25](#) Population dynamics (general)

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[bifurcation analysis](#); [codimension-three Bogdanov-Takens bifurcation](#); [structural sensitivity](#); [population models](#); [functional response](#)

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