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Algebraic independence of Mahler functions via radial asymptotics. (English) Zbl 1415.11104
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Summary: We present a new method for algebraic independence results in the context of Mahler's method. In particular, our method uses the asymptotic behavior of a Mahler function $f(z)$ as z goes radially to a root of unity to deduce algebraic independence results about the values of $f(z)$ at algebraic numbers. We apply our method to the canonical example of a degree two Mahler function; that is, we apply it to $F(z)$, the power series solution to the functional equation $F(z) - (1 + z + z^2)F(z^4) + z^4F(z^{16}) = 0$. Specifically, we prove that the functions $F(z)$, $F(z^4)$, $F'(z)$, and $F'(z^4)$ are algebraically independent over $\mathbb{C}(z)$. An application of a celebrated result of Ku. Nishioka then allows one to replace $\mathbb{C}(z)$ by \mathbb{Q} when evaluating these functions at a nonzero algebraic number α in the unit disc.

MSC:

[11J85](#) Algebraic independence; Gel'fond's method

Cited in 7 Documents

Keywords:

Mahler's method; algebraic independence; Mahler functions

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