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**The Eisenstein cocycle and Gross's tower of fields conjecture.** (English. French summary)

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Summary: This paper is an announcement of the following result, whose proof will be forthcoming. Let $F$ be a totally real number field, and let $F \subset K \subset L$ be a tower of fields with $L/F$ a finite abelian extension. Let $I$ denote the kernel of the natural projection from $\mathbb{Z}[\mathrm{Gal}(L/F)]$ to $\mathbb{Z}[\mathrm{Gal}(K/F)]$. Let $\Theta \in \mathbb{Z}[\mathrm{Gal}(L/F)]$ denote the Stickelberger element encoding the special values at zero of the partial zeta functions of $L/F$, taken relative to sets $S$ and $T$ in the usual way. Let $r$ denote the number of places in $S$ that split completely in $K$. We show that $\Theta \in I^r$, unless $K$ is totally real in which case we obtain $\Theta \in I^{r-1}$ and $2\Theta \in I^r$. This proves a conjecture of Gross up to the factor of 2 in the case that $K$ is totally real and $\#S \neq r$. In this article we sketch the proof in the case that $K$ is totally complex.

**MSC:**

11R42 Zeta functions and $L$-functions of number fields
11R80 Totally real fields



**Keywords:**

Stickelberger elements; Eisenstein cocycle; Gross's conjecture

**Full Text:** DOI

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