Krejčířík, David; Leonardì, Gian Paolo; Vlachopulos, Petr

The Cheeger constant of curved tubes. (English) Zbl 1415.49027

If $\Omega \subset \mathbb{R}^d$, $d \geq 1$, is an open connected set, then the Cheeger constant of $\Omega$ is the number $h(\Omega) = \inf_{S \subset \Omega} \frac{|\partial S|}{|S|}$, where the infimum is taken over all non-empty bounded sets $S \subset \Omega$, $|S|$ is the volume of $S$, and $|\partial S|$ is the perimeter of $S$. Any minimiser of $h$, if it exists, is called a Cheeger set of $\Omega$ and is denoted by $C_\Omega$. If $\Gamma$ is a closed smooth curve in $\mathbb{R}^d$ and $a$ is a positive number, then the set $\Omega_a = \{x \in \mathbb{R}^d; \text{dist}(x, \Gamma) < a\}$ is called a curved tube. $\Omega_a$ is said to be not overlapping itself if the map $\Gamma \times (0, a) \ni (q, t) \mapsto q + tN(q)$ induces a smooth diffeomorphism for any smooth normal vector field $N$ along $\Gamma$. In [Zbl 1247.28003], the first author and A. Pratelli showed that if $d = 2$, then $h(\Omega_a) = \frac{1}{a}$ and $C_{\Omega_a} = \Omega_a$.

In this paper the authors extend these results to the case of $d \geq 2$ and they prove that if $\Gamma$ is a closed smooth curve in $\mathbb{R}^d$ and a positive number $a$ is so small that $\Omega_a$ does not overlap itself, then $h(\Omega_a) = \frac{d-1}{a}$ and $C_{\Omega_a} = \Omega_a$. The second goal of this paper is to raise a challenging open problem about determining the Cheeger constant of tubular neighbourhoods of general submanifolds $M$ of $\mathbb{R}^d$. If $M$ is a spherical shell, then the authors show that for two positive radii $r < R$, the spherical shell $A_{r,R} = \{x \in \mathbb{R}^d; r < |x| < R\}$ is a minimal Cheeger set, and $h(A_{r,R}) = d\frac{R^{d-1} - r^{d-1}}{R^d - r^d}$.

Reviewer: Andrew Bucki (Edmond)

MSC:
49Q10 Optimization of shapes other than minimal surfaces
49Q15 Geometric measure and integration theory, integral and normal currents in optimization
49Q20 Variational problems in a geometric measure-theoretic setting
28A75 Length, area, volume, other geometric measure theory
35P15 Estimates of eigenvalues in context of PDEs
51M16 Inequalities and extremum problems in real or complex geometry

Keywords:
Cheeger constant; Cheeger set; curved tubes; tubular neighbourhoods of curves; spherical shells

Full Text: DOI arXiv

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.