Summary: The complex morphologies exhibited by spatially confined thin objects have long challenged human efforts to understand and manipulate them, from the representation of patterns in draped fabric in Renaissance art to current-day efforts to engineer flexible sensors that conform to the human body. We introduce a theoretical principle, broadly generalizing Euler’s elastica—a core concept of continuum mechanics that invokes the energetic preference of bending over straining a thin solid object and that has been widely applied to classical and modern studies of beams and rods. We define a class of geometrically incompatible confinement problems, whereby the topography imposed on a thin solid body is incompatible with its intrinsic (‘target’) metric and, as a consequence of Gauss’ Theorema Egregium, induces strain. By focusing on a prototypical example of a sheet attached to a spherical substrate, numerical simulations and analytical study demonstrate that the mechanics is governed by a principle, which we call the ‘Gauss-Euler elastica.’ This emergent rule states that despite the unavoidable strain in such an incompatible confinement—the ratio between the energies stored in straining and bending the solid may be arbitrarily small. The Gauss-Euler elastica underlies a theoretical framework that greatly simplifies the daunting task of solving the highly nonlinear equations that describe thin solids at mechanical equilibrium. This development thus opens possibilities for attacking a broad class of phenomena governed by the coupling of geometry and mechanics.

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